

# A new method to reduce frequency-temperature coefficient of sapphire loaded cavity for compact hydrogen masers

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**Abstract**—In order to reduce the size and weight of the hydrogen maser atomic clocks, some useful attempts and related research results about sapphire loaded cylindrical cavity for the hydrogen maser were reported in the Beijing Institute of Radio Metrology and Measurement. The fractional frequency stability of the order of  $10^{-15}$  over ten thousand seconds can be realized. However, due to large frequency-temperature coefficient in a single sapphire bulb in the cavity, further improvement of the stability in the compact hydrogen clock was restricted. In this work, we chose several small single-crystal chips of  $\text{SrTiO}_3$  with a large negative frequency-temperature coefficient to compensate the sapphire cavity. The permittivity of  $\text{SrTiO}_3$  is 300 along the [001] crystallographic direction at  $50^\circ\text{C}$ , which is about thirty times magnitude larger than that of sapphire (that is 9.4 along the  $z$ -axis of sapphire single crystal) at the same temperature. Thus, through theoretical calculation, finite element method simulation, and experiments, we have successfully performed the compensation of frequency-temperature coefficient in sapphire cavity using small  $\text{SrTiO}_3$  rings for chips. Based on theoretical calculation, the frequency-temperature coefficient in the  $\text{TE}_{011}$  mode of a sapphire cavity associated with two small rings of  $\text{SrTiO}_3$  can be reduced obviously. For instance, at  $50^\circ\text{C}$ , when the thickness of the  $\text{SrTiO}_3$  ring is 5 mm, the frequency-temperature coefficient can be reduced about five times, and quality factor can be kept at above 40000 synchronously. A sapphire loaded cavity and eight small compensated chips of  $\text{SrTiO}_3$  were prepared and a combined cavity was simulated by finite element method and measured by experiments. The simulation and the experimental results agree very well. In addition, experimental results show that frequency-

temperature coefficient in eight small  $\text{SrTiO}_3$  chips adhered to one endcap of sapphire bulb is much better than that in top and bottom of sapphire symmetrically.

## I. INTRODUCTION

The hydrogen maser was realized in 1960s by Kleppner and Ramsey et al. [1-3]. The hydrogen maser was mainly applied as a time reference due to its excellent performance in medium- and long-term frequency stability. However, the overall size is a handicap in a number of operational applications in the vicinity of the earth's surface and on board satellites although a hydrogen maser having a standard-size microwave cavity has been successfully launched on a ballistic trajectory [4]. Research programmes were thus started in order to develop smaller-size microwave cavities, still resonating at the hydrogen atom hyperfine frequency. A  $\text{TE}_{111}$  microwave cavity has been successfully used [5]. In other approaches, the microwave cavity has been loaded either by slotted metallic cylinders surrounding the storage bottle [6] or by a hollow cylinder made of a high-dielectric-constant material, such as alumina [7] or sapphire [8,9]. These designs have led to a drastic reduction in size of hydrogen maser.

The sapphire-loaded microwave cavity is very effective for reducing the size and weight of the hydrogen maser. The dielectric loss ( $\tan\delta$ ) of the sapphire is small enough so that the loaded quality factor ( $Q$ ) is only slightly affected. In fact, the  $Q$  is more affected by the skin depth of the cavity wall or the conductivity of the wall. The sapphire-loaded microwave cavity with the low loss factor and high dielectric constant can remain the high quality

factor  $Q$ , resulting in the high stability and accuracy characteristics of hydrogen maser. The fractional frequency stability of the order of  $10^{-15}$  over ten thousand seconds has been realized in compact hydrogen maser with sapphire microwave cavity in Beijing Institute of Radio Metrology and Measure (BIRMM, China) [8]. However, due to the effect of frequency-temperature coefficient in sapphire-loaded microwave cavity, further improving the frequency stability and control of the compact hydrogen maser was restricted.

Duo to the need of more sapphire filling when reducing the cavity volume further, the frequency-temperature coefficient of the sapphire microwave cavity is difficult to control. This is due to single sapphire materials with only a positive frequency-temperature coefficient filling the cavity. In this work, we choose a related single crystal of  $\text{SrTiO}_3$  with a large negative frequency-temperature coefficient to compensate the sapphire cavity, which can reduce the influence of cavity frequency with temperature in single sapphire material. The permittivity of  $\text{SrTiO}_3$  is 300 along the [001] crystallographic direction at  $50^\circ\text{C}$ , which is an order of magnitude larger than that of sapphire (that is 9.4 along the  $z$ -axis) at the same temperature. And,  $\text{SrTiO}_3$  is also a low-loss dielectric material ( $8 \times 10^{-4}$  at  $50^\circ\text{C}$ ). A new method with several small  $\text{SrTiO}_3$  chips sticking to the endcap of sapphire bulb is used to reduce frequency-temperature coefficient of microwave cavity in compact hydrogen maser through theoretical calculation, finite element method simulation (FEM), and experimental methods.

## II. THEORETICAL AND EXPERIMENTAL SET-UP

A low-loss  $\text{TE}_{011}$  mode resonant microwave cavity with sapphire bulb was used in the frequency-temperature compensated set-up, as shown in figure 1. The material of microwave cavity is titanium metal because of its non-magnetic characteristic and an excellent low thermal expansion coefficient. A cylindrical sapphire bulb is placed in centre of the titanium cavity. The crystallographic [0001] direction of the sing-crystal sapphire is aligned to the cylinder axis of the cavity. Eight small single-crystal  $\text{SrTiO}_3$  chips were stuck to the one endcap of sapphire bulb, their crystallographic [001] direction parallel to the cylinder axis of the microwave cavity, as shown in figure 1. Some former experimental results about the anisotropic permittivity, frequency-

temperature coefficient, dielectric loss coefficient and thermal expansion coefficient of sapphire and  $\text{SrTiO}_3$  are used in the calculation and simulation [10,11]. In order to find the right frequency-temperature compensated effects, different thickness of  $\text{SrTiO}_3$  chips were theoretical simulated and experimental tested.

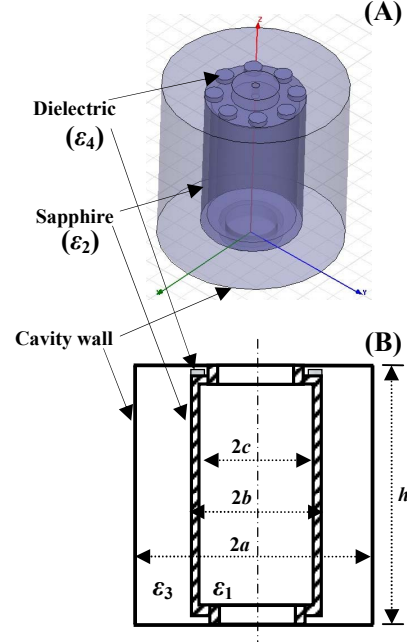


Figure 1 Configuration of cavity with temperature-compensated  $\text{SrTiO}_3$  single crystal loaded in one endcap of sapphire bulb for theoretical analysis (“ $a = 88 \text{ mm}$ ” and “ $h = 87.5 \text{ mm}$ ” are radius and height of microwave cavity, “ $b = 47.55 \text{ mm}$ ” and “ $c = 41.3 \text{ mm}$ ” are outer and inner radii of sapphire, respectively.  $\epsilon_1 = \epsilon_3 = \epsilon_0$  is the vacuum relative permittivity:  $\epsilon_0 = 1$ ,  $\epsilon_2$  is relative permittivity of sapphire:  $\epsilon_2 = 9.4$ , and  $\epsilon_4$  is relative permittivity of  $\text{SrTiO}_3$ :  $\epsilon_4 = 300$ ).

## III. THEORETICAL CALCULATIONS AND ANALYSIS

Ignoring the influence of temperature change to the thickness of  $\text{SrTiO}_3$ , the equation between temperature and frequency can be derived

$$\frac{1}{f} \frac{\partial f}{\partial T} = \frac{1}{f} \left[ \frac{\partial f}{\partial \epsilon_{11}} \frac{\partial \epsilon_{11}}{\partial T} + \frac{\partial f}{\partial \epsilon_{12}} \frac{\partial \epsilon_{12}}{\partial T} + \frac{\partial f}{\partial d} \frac{\partial d}{\partial T} + \frac{\partial f}{\partial h_1} \frac{\partial h_1}{\partial T} + \frac{\partial f}{\partial a} \frac{\partial a}{\partial T} + \frac{\partial f}{\partial h_2} \frac{\partial h_2}{\partial T} \right] \quad (1)$$

$d = b - c$ , substituting for  $(\partial f / \partial T)$  by  $(\Delta f / \Delta T)$  in Eq. (1) further, and others have some similar changes. The equation can be derived from the Eq. (1) as follows

$$\begin{aligned}\tau_f = & A_{\perp 1}\tau_{\perp 1} + A_{\perp 2}\tau_{\perp 2} + A_d\tau_{\alpha 1} \\ & + A_{h1}\tau_{\alpha 2} + A_a\tau_c + A_{h2}\tau_c,\end{aligned}\quad (2)$$

where

$$\begin{cases}\tau_f = \frac{\Delta f}{f \Delta T}; \\ \tau_{\perp 1} = \frac{\Delta \epsilon_{\perp 1}}{\epsilon_{\perp 1} \Delta T}; \tau_{\perp 2} = \frac{\Delta \epsilon_{\perp 2}}{\epsilon_{\perp 2} \Delta T}; \tau_{\alpha 1} = \frac{\Delta d}{d \Delta T}; \\ \tau_{\alpha 2} = \frac{\Delta h_1}{h_1 \Delta T}; \tau_c = \frac{\Delta a}{a \Delta T} = \frac{\Delta h_2}{h_2 \Delta T}; \\ A_{\perp 1} = \frac{\epsilon_{\perp 1}}{f} \frac{\Delta f}{\Delta \epsilon_{\perp 1}}; A_{\perp 2} = \frac{\epsilon_{\perp 2}}{f} \frac{\Delta f}{\Delta \epsilon_{\perp 2}}; A_d = \frac{d}{f} \frac{\Delta f}{\Delta d}; \\ A_a = \frac{a}{f} \frac{\Delta f}{\Delta a}; A_{h1} = \frac{h_1}{f} \frac{\Delta f}{\Delta h_1}; A_{h2} = \frac{h_2}{f} \frac{\Delta f}{\Delta h_2}.\end{cases}\quad (3)$$

In Eqs. (1) and (2),  $\tau_f$  is temperature coefficient of resonant frequency,  $\tau_{\perp 1}$ ,  $\tau_{\perp 2}$ , and  $\tau_c$  are temperature coefficient of sapphire, SrTiO<sub>3</sub>, and Ti cavity, respectively.  $\tau_{\alpha 1}$  and  $\tau_{\alpha 2}$  are the coefficient of the thermal linear radial and axial expansion coefficient of the dielectric and conductor. At 50°C and 1.42 GHz,  $\tau_{\perp 1}$  and  $\tau_{\perp 2}$  perpendicular to the *c*-axis of cavity is 90 ppm/K and 215 ppm/K for sapphire and SrTiO<sub>3</sub>, respectively,  $\tau_{\alpha 1}$  is 4.5 ppm/K;  $\tau_{\alpha 2}$  is 5.3 ppm/K; and  $\tau_c$  is 8.6 ppm/K.

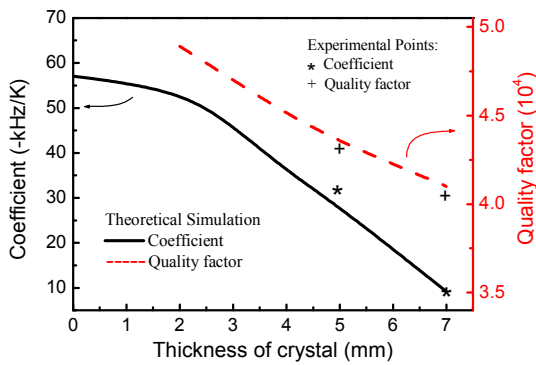


Figure 2 Calculated and experimental results of frequency-temperature coefficient and cavity quality factor change with thickness of SrTiO<sub>3</sub> crystal chips.

Frequency-temperature coefficient and quality factor of the sapphire resonator as a function of the thickness of

SrTiO<sub>3</sub> have been calculated and shown in figure 2. With increasing the thickness of SrTiO<sub>3</sub>, the frequency-temperature coefficient decreases and quality factor of microwave cavity synchronously.

#### IV. EXPERIMENTAL RESULTS AND DISCUSSION

We firstly confirm the frequency-temperature coefficient with only sapphire cavity and without SrTiO<sub>3</sub> dielectric. Because the actual hydrogen maser is operated at 50□, when choosing three temperature points 45□, 50□, and 55□, the cavity frequency can be measured as 1.420699 GHz, 1.420405GHz, and 1.420112 GHz, respectively. Thus, we can obtain that the frequency-temperature coefficient is -58.8 kHz/K, which is very close to the theoretically calculated results -57.06 kHz/K.

When the sapphire bulb combines with eight SrTiO<sub>3</sub> chips, we can obtain the frequency-temperature coefficients of microwave cavity in sapphire with different thickness of SrTiO<sub>3</sub> chips. The experimental results show in figure 2 and these data are very close to the simulation results.

The prototype of hydrogen maser atomic clocks with compensated microwave cavity and its Allan deviation have been shown in figure 3. The total volume of compact hydrogen maser is 95 L (48×34×58 cm).

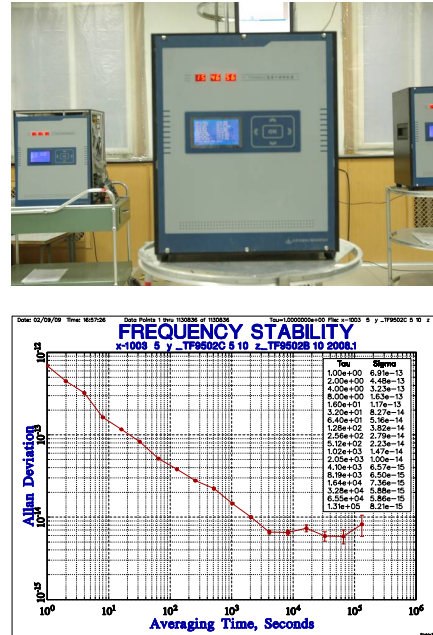


Figure 3 the prototype and performance of the compact hydrogen maser

## V. CONCLUSION

In conclusion, a distinct decrease of the frequency-temperature coefficient in a compact hydrogen maser atomic clock has been realized by calculation, FEM simulation and related experiment. Solution has been implemented by associating sapphire with another dielectric material  $\text{SrTiO}_3$ . The experimental results agree well with that of calculation and simulation. This works have guided us to advance the compact hydrogen maser further.

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